CIRCULAR.

WITH the view of establishing, in the operations of the Corps, uniformity and facility in its astronomical observations, particularly of such as are intended for determinations of longitudes, my attention has been frequently drawn to the method, designated in works on astronomy as that of Lunar Culminations, as depending upon a class of phenomena of frequent occurrence, of easy calculation, and affording correct results; and, desirous of obtaining the best information from the most informed and most practical minds on these subjects, application was made to Professor W. H. C. BARTLETT, for a detailed account of the course of instruction pursued by him on this subject at the U. S. Military Academy. He has supplied me with the subjoined extract from his course of instruction, illustrated by recent observations made by himself.

The officer will perceive, that as the only observations to be made are those necessary to determine the difference of right ascension between the bright limb of the moon and certain fixed stars, previously designated in the English Nautical Almanac, under the head of "Moon-culminating Stars;" or, in other words, to note the indications of a time-piece, whose *rate* simply is known, when the border of the Moon and certain stars pass the line of collimation of a Transit adjusted but *approximatively* to the meridian, and that as these observations are within the compass of a Chronometer and Transit Instrument, or even a Theodolite, he will always be able to determine the longitude of a station. And from the frequent occurrence of the phenomena observed, he will be able to repeat his observations as often as desirable, and thus produce results deserving of confidence and creditable to his reputation.

The Colonel of the Corps has no desire to limit the operations of his officers to the class of phenomena herein designated. They can try others, but he directs that in all cases, the course herein designated be pursued, and that results be duly and regularly transmitted to the Bureau.

BUREAU CORPS OF TOPOGRAPHICAL ENGINEERS,
JANUARY 31, 1845.

J. J. Abert
Col. Corps T. E.

LONGITUDE BY LUNAR CULMINATIONS.

1. The principle of this method, which is due to Mr. Francis Baily, of London, and M. Nicolai, of Manheim, is this:—If the distance through which the Moon moves in right ascension, during the interval between her passages over any two meridians be known, the interval itself, and therefore the difference of longitude of the meridians will become known, from the known rate of motion during any other interval.

To find the Moon's change of right ascension while passing from one meridian to the other:

Let S', denote the true *sidereal* time of the Moon's centre passing the meridian of a place;

H', the hour indicated by the time-piece at the instant of the bright limb's passing the same meridian;

e', the error of the time-piece from true *sidereal* time;

i', the error of the Transit in time, for altitude equal to that of the Moon;

r', the semi-diameter of the Moon;

δ', the declination of the Moon;

Then will

$$S' = H' + e' + i' \pm \frac{r'}{15 \cos \delta'} \quad . \quad . \quad . \quad (1)$$

in which the upper sign is to be taken when the bright limb of the Moon is turned to the west, or when the Moon is between new and full; and the lower, when the bright limb is turned to the east, or the Moon is between the full and new.

Let s', denote the *sidereal* time of a star's passing the same meridian just before or after the Moon, the star having very nearly the same declination as the Moon;

H'', the hour indicated by the time-piece;

e'', the error of the same at the instant of passage.

Then, as the declinations of the Moon and star are the same, or nearly so, the error of the Transit in time will be the same as for the Moon, and we shall have

$$s' = H'' + e'' + i' \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and, subtracting this from Equation (1,) we have,

$$S' - s' = H' - H'' + e' - e'' \pm \frac{r'}{15 \cos \delta'} \quad . \quad . \quad . \quad (3)$$

Make $e'' = e' \pm \varphi$, and Equation (3) reduces to

$$S' - s' = H' - H'' \mp \varphi \pm \frac{r'}{15 \cos \delta'} \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

in which φ obviously represents the acceleration or retardation of the time-piece over *sidereal* time, during the interval between the Moon and star passing the middle wire of the Transit; and making

$$H' - H'' \mp \varphi = t' \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

we have

$$S' - s' = t' \pm \frac{r'}{15 \cos \delta'} \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

in which t', represents the time-piece interval between the observations, *corrected for the* RATE *of the instrument.*

For a second meridian to the west, we have, since the star does not change its right ascension sensibly during a few hours, in like manner,

$$S'' - s' = t_{\prime} \pm \frac{r_{\prime}}{15 \cos \delta_{\prime}}$$

and, subtracting Equation (6) from this, we get

$$\triangle = S'' - S' = t_{\prime} - t' \pm \frac{1}{15} \left\{ \frac{r_{\prime}}{\cos \delta_{\prime}} \mp \frac{r'}{\cos \delta'} \right\} \quad . \quad . \qquad (7)$$

in which $\triangle$, is the change in right ascension of the Moon, during the interval of her passage from one meridian to the other.

2. Let a', denote the true right ascension, in time, of the Moon at the instant of her passing any assumed meridian; $a_{\prime}$, her right ascension at the time of passage over any other meridian to the west; and represent by c' and $c_{\prime}$, the corresponding indications of a perfectly accurate sidereal time-piece at some first meridian; then will the arc of the Equator, *in time*, passed over by the western meridian in the time $c_{\prime} - c'$, be equal to the difference of longitude of the assumed meridians, increased by $a_{\prime} - a'$; this latter being the motion of the Moon in right ascension while she is passing from one meridian to the other. And if x, denote the arc of the Equator, in time, passed over by a meridian while the Moon is changing her right ascension by $\triangle$, we shall have

$$a_{\prime} - a' : c_{\prime} - c' :: \triangle : x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (A)$$

or

$$x = \triangle \, \frac{c_{\prime} - c'}{a_{\prime} - a'} \, ;$$

and denoting the difference of longitude corresponding to the change $\triangle$, by L, we shall have

$$L = \triangle \, . \frac{c_{\prime} - c'}{a_{\prime} - a'} - \triangle = \triangle \, . \frac{\overline{c_{\prime} - c'} - \overline{a_{\prime} - a'}}{a_{\prime} - a'} \quad . \quad . \quad (8)$$

but $\overline{c_{\prime} - c'} - \overline{a_{\prime} - a'}$, is the difference of longitude of the assumed meridians, and $a_{\prime} - a'$ is the Moon's change in right ascension while passing from one of these meridians to the other. Denoting the first by l, and the second by a, Equation (8) becomes

$$L = \triangle \, . \frac{l}{a}$$

or

$$L = \left[\, t_{\prime} - t' \pm \frac{1}{15} \left\{ \frac{r_{\prime}}{\cos \delta_{\prime}} \mp \frac{r'}{\cos \delta'} \right\} \right] . \frac{l}{a} \quad . \quad . \quad . \quad (9)$$

If what has been said of the centre of the Moon, be understood of the bright limb, $r_{\prime}$ and r', in this equation, will be zero, and we finally have

$$L = (\, t_{\prime} - t' \,) \, . \frac{l}{a} \quad . \quad . \quad . \quad . \quad (10)$$

3. The proportion (A,) is only true on the supposition that the Moon's motion in right ascension is uniform; which is by no means the case. The error may, however, be rendered inappreciable in two ways, viz:

1st. By taking the *assumed* meridians, whose difference is l, nearly coincident with those whose difference of longitude is to be determined; or,

2d. By taking the *assumed* meridians distant from each other one hour, and equally distant from a meridian midway, or very nearly so, between those whose difference of longitude is to be found.

In either case, the approximate difference of longitude must be known.

In the Nautical Almanac are given the right ascension of the bright limb of the Moon, and that of one or more stars near the Moon's parallel of declination, and not differing from her much in right ascension, for every day in the year, at the instant of passing the upper and lower meridian of Greenwich. These may be regarded as the results of observations on that meridian, and used correspondingly with those of observations made at any other meridian. In a separate column, and immediately opposite the right ascension of the Moon at each transit, her variation in right ascension during the interval between her transits over two meridians, equally distant from that of Greenwich, and one hour from each other, is also given.

Knowing, then, the true right ascension of the Moon's bright limb when on the upper and lower meridian of Greenwich, her true right ascension, when on any other meridian, is easily found by the method of interpolation; or, knowing the motion of the Moon's limb in right ascension while between two meridians, separated by one hour, and equally distant from that of Greenwich, its motion, while between two other meridians separated by the same interval and equally distant from a meridian midway between that of Greenwich and the one whose longitude is required, is found by the same method.

The ordinary series for interpolation is

$$a = V + n \triangle^1 + \frac{n\,(n-1)}{1\,.\,2} \,.\, \triangle^2 + \frac{n\,.\,(n-1)\,(n-\frac{1}{2})}{1\,.\,2\,.\,3} \,.\, \triangle^3 + \&c.$$

in which a, is the required value, V, the next preceding tabular value, n, the ratio of the tabular interval to that between the last tabular and required value; $\triangle^1$, $\triangle^2$, $\triangle^3$, &c. the first, second, third, &c. differences.

Performing the operations indicated in the several terms, and arranging the series according to the ascending powers of n, we have

$$a = V + A\,n + B\,n^2 + C\,n^3\,;$$

in which,

$$A = \triangle^1 - \tfrac{1}{2}\triangle^2 + \tfrac{1}{12}\triangle^3$$
$$B = \tfrac{1}{2}\triangle^2 - \tfrac{1}{4}\triangle^3$$
$$C = \tfrac{1}{6}\triangle^3$$

The tabular interval in the case before us is 12^{h}, being the difference of longitude between the upper and lower meridian of Greenwich; and if m, be the longitude of the middle meridian before referred to, then will

$$n = \frac{m}{12}\,,$$

and we finally have

$$a = V + A\,.\left\{\frac{m}{12}\right\} + B\left\{\frac{m}{12}\right\}^2 + C\,.\left\{\frac{m}{12}\right\}^3.$$

4. The great merit of this method is its simplicity, the frequency with which it may be employed, and its independence of all instrumental error, as is readily seen by a reference to Equations (5,) (6,) and (10,) from which the error of the Transit and time-piece, the only instruments employed, are eliminated.

5. The accuracy is increased when observations are *actually made* at the *two* stations, as in that case the errors of the Lunar Tables, and those of nutation and aberration are eliminated.

The elements of the Nautical Almanac give very good approximations; and these may be corrected by preserving the observations till the corresponding observations at Greenwich, or some other Observatory whose position is accurately known, may be obtained. These observations are published annually.

FIRST METHOD,

In which the assumed meridians are nearly coincident with those whose difference of longitude is to be found.

EXAMPLE.

Longitude of West Point, by Lunar Culminations, Feb. 18, 1845.

			h.	m.	s.		h.	m.	s.
Feb. 18.		*Nautical Almanac.*							
" "	ζ	Geminorum,	6	54	57.41				
" "	δ	"	7	10	54.36				
" "	☽	W. Limb,					7	27	47.66
" "	ζ	Cancri,	8	03	21.44				
			3)22	09	13.21		7	23	04.403
						$t' =$	0	04	43.257
" "		*West Point.*							
" "	ζ	Geminorum,	6	54	47.75				
" "	δ	"	7	10	38.97				
" "	☽	W. Limb,					7	38	06.76
" "	ζ	Cancri,	8	03	06.11				
			3)22	08	26.83		7	22	48.943
							0	15	17.817
		Rate of Clock, $+ 3^s$							—.0318
						$t_{,} =$	0	15	17.785
						$t_{,} - t' =$	0	10	34.528

Interpolation.

	h.	m.	s.	m.	s.		s.	
February 17, L. C.	7	01	56.27					
				25	51.39			
" 18, U. C.	7	27	47.66				-10.21	
				$\triangle' =$ 25	41.18	$\triangle^2 = \frac{1}{2}\Sigma$		$\triangle^3 = -.25$
" 18, L. C.	7	53	28.84				-10.46	
				25	30.72			
" 19, U. C.	8	18	59.56					

$$A = 25^{m}\ 41^{s}.18 + 05^{s}.1675 - .02083 = 25^{m}\ 46^{s}.32667$$
$$B = -\ 05.1675 + 0.0625 \quad . \quad . \quad = -\ 5.105$$
$$C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad = -\ 0.0416$$

Approximate longitude of West Point, $4^h\ 55^m\ 50^s = 17750^s = m$

Nos.	Logs.	Nos.	Logs.	Nos.	Logs.
m	4 . 2491984	m^2 . .	8 . 4983968	m^3 . . .	12 . 7475952
$a. c.$. 12^h	$\bar{5}$. 3645163	$(12^h)^2$ . .	$\overline{10}$. 7290326	$(12^h)^3$ . . .	$\overline{14}$. 0935489
	$\bar{1}$. 6137147		$\bar{1}$. 2274294		$\bar{2}$. 8411441
A	3 . 1893022	B . . .	0 . 7079957	C	$\bar{2}$. 6190933
$635^s.356$ . . .	2 . 8030169	$-0^s.861837$	$\bar{1}$. 9354251	$-^s.00288561$	$\bar{3}$. 4602374

$$
\begin{array}{rl}
 & 635.356 \\
- & 0.861837 \\
- & 0.008856 \\
\hline
 & 634.485 = a
\end{array}
$$

$$L = (t_{,} - t')\frac{l}{a} = 634^s.528 \times \frac{17750^s}{634^s.485}$$

Nos.		Logs.
634.528	. . . .	2 . 8024508
17750.	. . . .	4 . 2491984
$a\ c$ 634.485	. . . .	$\bar{3}$. 1975786
Long. W. $= 4^h.\ 55^m.\ 51^s.\ 3 = 17751^s.\ 3$	. .	4 . 2492278

SECOND METHOD,

In which the Middle Meridian is used.

EXAMPLE I.

Longitude of West Point, by Lunar Culminations, Feb. 18, 1845.

Feb. 18. — *Nautical Almanac.*

		h.	m.	s.		h.	m.	s.
" "	ζ Geminorum,	6	54	57.41				
" "	δ "	7	10	54.36				
" "	☽ W. Limb, .	.	.	.	. . . .	7	27	47.66
" "	ζ Cancri, .	8	03	21.44				
		3)22	09	13.21	. .	7	23	04.403
					$t' =$	0	04	43.257

Feb. 18. *West Point.*

		h.	m.	s.		h.	m.	s.
	ζ Geminorum,	6	54	41.75				
" "	δ "	7	10	38.97				
" "	☽ W. Limb,	.	.	.	. . .	7	38	06.76
" "	ζ Cancri, .	8	03	06.11				
		3)22	08	26.83	. .	7	22	48.943
						0	15	17.817
	Rate of Clock, $+ 3^s$							—.03
					$t_, =$	0	15	17.787
					$t_, - t' =$	0	10	34.53

Interpolation.

		s.				
February 17,	L. C.	129.68				
			. . . — , 81			
" 18,	U. C.	128.87			$\Big\{$ — , 07	
			. . $\triangle^1 =$ — , 88	$\triangle^2 = \frac{1}{2} \Sigma$		$\triangle^3 =$, 09
" "	L. C.	127.99			$\Big\{$, 02	
			. . . — , 86			
" 19,	U. C.	127.13				

$$\begin{aligned} A &= -\,.88 \quad + .0125 + .0075 = -\ 0^s.86 \\ B &= -\,.0125 - .0225 \quad . \quad . \quad = -\ 0.035 \\ C &= \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad\ \ 0.015 \end{aligned}$$

	h.	m.	s.
Approximate longitude of West Point from Greenwich,	4	55	50
$m =$ . . .	2	27	55

Nos.	Logs.	Nos.	Logs.	Nos.	Logs.
A . .	$\overline{1}$. 9344985	B .	$\overline{2}$. 5440680	C .	$\overline{2}$. 1760913
m . .	3 . 9481684	m^2 .	7 . 8963368	m^3 .	11 . 8445052
a c 12^h . .	$\overline{5}$. 3645163	$(12^h)^2$.	$\overline{10}$. 7290326	$(12^h)^3$.	$\overline{14}$. 0935489
s. — 0.1766 .	$\overline{1}$. 2471832	— 0^s. 001477	$\overline{3}$. 1694374	0^s. 00013006	$\overline{4}$. 1141454

— 0.1766
— 0.001477
0.000130

— 0.177947
128.87

128.692053 = a

$$L = (t_, - t')\,\frac{l}{a} = 634^s.53 \times \frac{3600}{128.692}$$

Nos.		Logs.
634.53	. . . .	2 . 8024522
3600.	. . . .	3 . 5563025
128.692	. . . .	$\bar{3}$. 8904485
Long. W. = 4^h 55^m 50.2^s = 17750.2^s	. . . . .	4 . 2492032

EXAMPLE II.

Longitude of West Point, from ACTUAL *observations of Moon Culminations at Greenwich and West Point.*

1836. February 25. Greenwich, (See Gr. Observ. 1836.)

		h.	m.	s.		m.	s.
☽	.	5	15	54.18			
H. Geminorum	. .	5	54	09.43	. . $t' =$	38	15.25

" West Point, *with a very indifferent transit instrument.*

☽	.	5	26	31.2			
H. Geminorum	. .	5	53	43.8	$t_{\prime} =$	27	12.60

$t' - t_{\prime} = 11 \; . \; 02.65 = 662.65^s$

Interpolation.

February 24, L. C. 131.39^s
" 25, U. C. 133.93
" " L. C. 136.18
" 26, U. C. 137.99

$\triangle^1$: 2.54^s, 2.25, 1.81 . . $\triangle^1 = 2.25$ . . $\triangle^2 = \frac{1}{2}\Sigma \begin{cases} -0.29^s \\ -0.44 \end{cases}$ $\triangle^3 = -0 , 15$

$$A = 2.25 + 0.1825 - 0.0125 = 2.42^s$$
$$B = -0.1825 + 0.0375 \; . \; . \; . = -0.145$$
$$C = - \; . \; . \; . \; . \; . \; . \; . \; -0.025$$

Approximate longitude of West Point from Greenwich, 4^h 55^m 50^s

$m =$ . . . 2 27 55

Nos.	Logs.	Nos.	Logs.	Nos.	Logs.
A . .	0 . 3838154	B .	$\bar{1}$. 1613680	C .	$\bar{2}$. 3979400
m . .	3 . 9481684	m^2 .	7 . 8963368	m^3 .	11 . 8445052
$a\ c\ 12^h$ . .	$\bar{5}$. 3645163	$(12^h)^2$.	$\overline{10}$. 7290326	$(12)^3$.	$\overline{14}$. 0935489
+ 0.497^s .	$\bar{1}$. 6965001	— 0^s.0061 .	$\bar{3}$. 7867374	— 0^s.000216 .	$\bar{4}$. 3359941

+ 0.497^s
— 0.0061
— 0.0002

0.4907
133.93

134.4207 . . . . . . $= a$

$$L = (t_{,} - t')\frac{l}{a} = 662^{s}.65 \times \frac{3600^{s}}{134.4207}$$

Nos.		Logs.
662 . 65	. . . . . .	2 . 8212842
3600 .	. . . . . .	3 . 5563025
a c 134 . 4207	. . . . . .	$\bar{3}$. 8715340
Long. W. = 4h. 55m. 46.8s. = 17746.8s.	. .	4 . 2491207

Clock rate — $0^{s}.06$ per hour was neglected.

REMARKS.

It frequently happens that the Moon cannot be observed on the middle wire, in which case she is far enough from the meridian to have a sensible parallax in right ascension; and as it may be very desirable not to lose the observation, this parallax must be computed and applied to the apparent hour angle from the middle wire, which is supposed to be nearly coincident with the meridian.

Denoting this parallax in right ascension by p, the horizontal parallax by $\tilde{\omega}$, the latitude of the place of observation by φ, and the true declination of the Moon by δ, we have from the ordinary series for the parallax in right ascension, neglecting the terms after the first, which would in this case be insignificant,

$$p = \theta \sin \tilde{\omega} \cos \varphi \sec . \delta ,$$

in which θ, is the hour angle, or equatorial interval in sidereal time from the lateral wire on which the Moon is observed to the central wire; so that, at the instant of observation, the actual distance of the Moon's limb from the central wire is

$$\theta - \theta \sin \tilde{\omega} . \cos \varphi \sec . \delta ,$$

and the reduction to meridian or middle wire will be

$$\pm \frac{\theta}{\cos \delta} . \frac{1 - \sin \tilde{\omega} . \cos \varphi . \sec . \delta}{1 - 0.00277\, m} ,$$

in which m, is the motion of the Moon in right ascension in one day, expressed in degrees. The upper sign is to be used when the observation is on a wire before, and the lower, after the middle wire.

7. It also often happens that two observers do not use the same number of wires, or if so, that the same stars are not observed at the same number. Such observations are not of equal weight. To find the relative value with which such observations should enter into the final determination, Professor Gauss has given the following formula, deduced from the principle of least squares.

Let the number of wires on which the Moon is observed at one place be denoted by n, and at the other by n'; and let the number of wires at which the stars are observed at the first place be a, b, c, &c. and at the other, be a', b', c', &c. Make

$$\frac{n\, n'}{n + n'} = \lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

$$\frac{a\, a'}{a + a'} = \alpha, \quad \frac{b\, b'}{b + b'} = \beta, \quad \frac{c\, c'}{c + c'} = \gamma, \text{ \&c.} \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

$$\sigma = \alpha + \beta + \gamma + \text{\&c.} \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

Then if W denote the weight of each day's comparison, will

$$W = \frac{\sigma\, \lambda}{(\sigma + \lambda)\, z^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

in which z is the same as $\frac{l}{a}$ in Equation (10); and for the weight of the result of all the comparisons, we have

$$\Sigma\, W = \Sigma \frac{\sigma\, \lambda}{(\sigma + \lambda)\, z^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

in which Σ expresses the sum.

Let e denote the probable error of observation, and E the probable error of the final result, then will

$$E = \Sigma \frac{e}{\sqrt{\Sigma \frac{\sigma\, \lambda}{(\sigma + \lambda)\, z^2}}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

The longitude of West Point, as determined from very imperfect measurements of the chord of the Solar Cusps, during the Solar Eclipse of the 9th of December, 1844, is

$$4^{h} . 55^{m} . 43^{s} . 6 .$$

The sun was just in the horizon when the measurements terminated.

This method of measuring the chord of the Solar Cusps cannot be too highly recommended. The chord towards the beginning and end of the Eclipse varying very rapidly, affords the means of finding the local time, corresponding to a determinate apparent distance between the centres of the Sun and Moon, with great accuracy; and avoids the risk of losing the observation of simple contact, the watching for which is painful and tedious. Besides, the observations may be multiplied at pleasure, thus diminishing the chances of error, by a system of checks, and giving the advantages of a general mean.